Bachelor in Mathematics

Titles, contents and timetable

Escuela Técnica Superior de Ingeniería Informática

Universidad Rey Juan Carlos

Contents

Prologue

This document contains information about titles, contents and timetable of subjects taught in the Bachelor in Mathematics, offered at Rey Juan Carlos University, Escuela Técnica Superior de Ingeniería Informática. This information pretends to be helpful to international students interested in visiting our University.

Contents in this document referred to subjects taught during course 2021-2022. More information can be consulted in

`https://www.urjc.es/estudios/grado/1245-matematicas`.

1

First Course

1.1 First Semester

1.1.1 Introduction to Programming

Introduction. Basic concepts in programming. Programming languages. How do computers represent information. Programming environments. An overview of our programming environment. Command line interface. Variables and assignment instruction. Primitive data types and expressions. Predefined functions and methods. Modules of interest. Our first program. Program structure. Input/output. Readability. Control Structures. Introduction. Selection instructions or conditional sentences. Iteration instructions or iterative sentences. Correction and debugging of control structures. Subprogramming Introduction. Using functions. Defining functions. Function calls in detail. Designing programs using functions. Recursion. Recursive functions. Data structures Introduction. One-dimensional arrays. Two-dimensional arrays. Algorithms on arrays. Lists. Stack, queue and array behavior. Dictionaries. Files 5.1 Introduction. Files.

6 ECTS credits.

1.1.2 Linear Algebra

Systems of linear equations, matrices and determinants, vector spaces, linear maps, eigenvalues and eigenvectors.

6 ECTS credits.

1.1.3 Logic

Sets, relations and functions. Propositional logic: syntax, semantics and proof theory. Predicate logic: syntax, semantics and proof theory.

6 ECTS credits.

1.1.4 Discrete Mathematics

Foundations and mathematical induction: Inductive and deductive model. Basic proof techniques. Mathematical induction and complete induction methods. Combinatorics: Set theory and counting techniques. Variations, permutations and combinations (with and without repetitions). Integer and modular arithmetic: Basic properties of natural and integer numbers. Integer divisibility and factorization. Greater common divisor. Linear diophantine equations. Congruence relations, modular arithmetic and groups. Linear congruence equations and systems. Applications: control digits and cryptography. Introduction to Graph theory : Basic concepts and models. Graph isomorphisms. Matrix representations of graphs. Paths, cycles and connected graphs. Eulerian and Hamiltonian graphs. Algebraic and combinatorial tools in graph theory.

6 ECTS credits.

1.1.5 Biological Fundamentals

Basic concepts: Exploring life Basic concepts of Cell Biology. Basic concepts of Biochemistry and Genetics. Photosynthesis. The origin of life and species. Basic concepts of Zoology. Basic concepts of Botany. Introduction to Ecology and Biosphere. Ecology of populations and communities. Ecology of ecosystems Applications. Mathematical models and their importance in Biology. Introduction to Bioinformatic. Computing advances inspired by Biology: computational architectures based on biological systems. Advances in robotics inspired by Biology: robots based on the structure and operation of plants and animals

6 ECTS credits.

1.2 Second Semester

1.2.1 Physical Fundamentals

Review of vectors. Kinematics of a particle. Dynamics of a particle. Gravitation. Dynamics of a system of particles. Work and energy. Electricity. Direct current. Theory of electric circuits. Analysis of linear circuits

6 ECTS credits.

1.2.2 Calculus

Real numbers, sequences and series. Real numbers. Sequences of real numbers. Series of real numbers. Real functions of real variable. Limits and continuity. Derivation and applications. Riemann integration. Sequences and series of functions.

6 ECTS credits.

1.2.3 Algebraic Structures

Rings. Ideals and quotient rings. Domains and fields of fractions. Isomorphism theorems. Polynomial rings. Irreducibility criteria for polynomials. Introduction to field theory. Groups. Lagrange's theorem. Homomorphisms, normal subgroups and quotient groups. Isomorphism theorems. Symmetric group. Group representations. Group actions, orbits, stabilizers and group quotients by group actions.

6 ECTS credits.

1.2.4 Probability

Random experiments. How to Calculate Probability. Random variables. Random vectors. Stochastic convergence.

6 ECTS credits.

1.2.5 Ethics, Legislation and Data Protection

Ethics, Legislation and data protection is a subject taught in the Degree in Mathematics in the second semester of the first year. It will examine the relationship between ethics and deontology and the profession of mathematician, as well as the constitutional treatment of the information, together with the basic legal principles of the legal system and the typology of the spanish legislation present at national and Community level, which concerns the issue of data protection. The objective to be achieved is that students know and understand the standards and regulation, both national as international, applicable in the field of the protection of personal data in the development of ICT, matter of especially sensitive in the era of development of the digital society.

6 ECTS credits.

2

Second Course

2.1 First Semester

2.1.1 Formal Languages

Preliminaries. Introduction. Formal languages. Grammars. Automata. The Chomsky hierarchy. Regular languages. Finite automaton: deterministic finite automaton, non deterministic finite automaton, equivalence of finite automaton and regular grammar. Regular expressions: definitions, equivalence of finite automaton and regular expressions. Properties of regular languages: operations on regular languages, decision algorithms, Pumping lemma. Context-free languages. Pushdown automaton. Definitions. Equivalences. Properties of context-free languages. Operations. Pumping lemma for context-free languages. Decision Algorithms. Some Applications. Recursively enumerable languages. Turing Machines. Definitions. Computable functions. Equivalences. Some applications.

6 ECTS credits.

2.1.2 Affine Geometry

Bilinear and sesquilinear forms. Euclidean space. The affine space. Affine Euclidean space. Conics and quadrics.

6 ECTS credits.

2.1.3 History of Science and Maths

Science and Ancient age: classic science. Science and Middle Ages: Islamic empire and European middle ages. Renaissance. Astronomy revolution: XVII century and the Age of Enlightenment. XVIII and XIX century: Industrial Revolution and Positivism. Maths revolution: hopes of the XX century and crisis. Technology and new perspectives.

6 ECTS credits.

2.1.4 Programming Methodology

Introduction. Overview of the Java language. Programming environment, data types, expressions, types of statements, input / output, functions, ... Object Oriented Programming (OOP). Principles of object-oriented design. UML. Class Diagrams. Other UML Diagrams. Fundamentals of Object Oriented Programming. Objects, classes, methods, attributes, messages, utility classes. Inheritance and polymorphism. Superclasses and subclasses, abstract classes, interfaces. Other characteristics of Object Oriented Programming (OOP). Genericity, exceptions, collections, input / output, introduction to event programming and graphical interfaces.

6 ECTS credits.

2.2 Second Semester

2.2.1 Statistics Methods for Management and Research

Linear Mathematical Optimization. Discrete optimization. Introduction to multicriteria analysis. Introduction to optimization under uncertainty.

6 ECTS credits.

2.2.2 Vector Analysis I

Functions of several variables. Continuity, definitions, limits. Differential and linear aproximation: partial derivative, differential, gradient, Jacobian, linear approximation, Chain rule, Implicit function, Inverse function, Change of variables, Higher order derivatives, Mean value theorem. Physical and geometric interpretations. Differentiable Functions of several variables: Taylor series, local approximations (linear, quadratic). Physical and geometric interpretations. Extrema: constraint extrema. Physical and geometric interpretations.

6 ECTS credits.

2.2.3 Topology

Topological spaces: notions about sets; functions, families and collections; cardinality of sets: properties of numerability. Topology: open and closed sets. Cofinite topology. Trivial topology and discrete topology. Euclidean topology. Basis of a topology: basis of a given topology. Subsets in a topological space: axioms of Separation; adherence; interior; Accumulation points and isolated points; Connected spaces. Construction of topological spaces: subspaces; topological product space; Initial and final topologies. Homeomorphisms: Homeomorphisms; Non homeomorphic spaces. Continuous functions: Continuous functions; Theorem of the intermediate value; Compacity; Normal spaces and extension of functions; Homotopy. Introduction to the fundamental group. Metric spaces: Metric spaces; Convergent sequences; Completeness. Contractions.

6 ECTS credits.

2.2.4 Chemical Fundamentals

Object and scope of Chemistry. Interrelation with the rest of the sciences: development technological. Chemistry and the information society. Basic concepts and fundamental calculations in Chemistry. Microscopic structure. Chemistry of atoms and molecules. Groups Theory. Experimental characterization. Measurements in Chemistry. States of aggregation. Description, equations of state. Chemical kinetics. Formal kinetics of chemical processes. Laws and mechanisms. Thermodynamics and equilibrium. Thermochemistry and chemical equilibrium.

6 ECTS credits.

2.2.5 Advanced Algebraic Structure

Group Theory (permutation groups, group actions, Sylow theorems, Solvable and Nilpotent groups). Galois Thery (Cyclotomic Polynomials. Roots of unity. Field extensions. Splitting fields. Galois Correspondence).

6 ECTS credits.

3

Third Course

3.1 First Semester

3.1.1 Data and Information Models

Database Foundation. Information Systems and Databases. File Systems versus Database Managment Systems. Data Models. Data model definition Entity/Relationship model. Conceptual modeling. Relational model. Database Design. Database Logical Design. Normalization theory. Database Implementation. SQL: Definition, Manipulation and Control languages.

6 ECTS credits.

3.1.2 Vector analysis II

Integrals of several variables. Differential forms. Stokes theorem.

6 ECTS credits.

3.1.3 Curves and Surfaces

Curves in space. Frenet thriedron. Curvature. Surfaces in space. First and second fundamental forms. Sectional and Gauss curvatures. Geodesics.

6 ECTS credits.

3.1.4 Ordinary Differential Equations

Introduction to ordinary differential equations. First order differential equations. Linear differential equations. Systems of first order differential equations with constant coefficients. Examples in models from physics, chemistry, biology etc. Existence and uniqueness of solutions to systems of differential equations and qualitative theory

6 ECTS credits.

3.1.5 Mathematical Statistics

Descriptive Statistics. Sampling methods. Data reduction. Properties of statistics: sufficient statistics, minimal sufficient statistics. Likelihood function. Point estimation. Methods to find estimators. Methods to evaluate estimators: unlikelihood, minimum variance, mean square error, consistency, robustness. Interval estimation. Methods for finding interval estimators. Iinterval estimators: Confidence intervals, Bayesian approximation. Methods to evaluate interval estimators: size and coverage probability. Hypothesis testing. Methods for finding hypothesis tests: likelihood tests. Methods to evaluate statistical tests: error probabilities and power function, most powerful tests.

6 ECTS credits.

3.2 Second Semester

3.2.1 Algorithm Design and Analysis

Introduction: basic concepts and recursion process Efficiency analysis: Efficiency analysis I: asymptotic study Efficiency analysis II: iterative and recursion algorithms. Algorithm design: Divide and conquer algorithm. Greedy algorithm. Backtracking.

6 ECTS credits.

3.2.2 Software Modelling

Introduction to Software Engineering. Introduction to Software Engineering. Software Engineering Workflows. Software Requirements. . Requirements Elicitation. Use Case Modelling. Requirements Analysis and Specification. Modelling of Analysis. Software Design. Modelling Design. Design Principles. Principles of Software Modelling. The Unified Modelling Language. Introduction to Software Processes. Introduction to Software Process Models. The Unified Software Development Process.

6 ECTS credits.

3.2.3 Computational Geometry

Polygons and polyhedra, convex hulls, Delaunay triangulations, Voronoi diagrams and duality. Introduction to topological manifolds. Description and classifications of compact surfaces. Euler characteristics. Arcs on surfaces. Introduction to the fundamental group of surfaces. The Seifert-van Kampen Theorem.

6 ECTS credits.

3.2.4 Partial Differential Equations

Introduction to Partial Differential Equations (PDE). First and Second order equations. Classification. Initial Value Problems and Boundary Value problems. Definition. The first order wave equation. Cauchy and Initial Value Problems. Geometrical interpretation. Potential fields. Stream Function. The method of characteristics. Lagrangian Coordinates. PDE systems of first order hiperbolic equations. The heat equation in bounded domains. Some variants. Modeling transport phenomena. Diffusion, Convection, Reaction, Absorption and Forcing. Fourier Analysis. 1D-Resolution. The second order Wave equation in bounded domains. Modeling friction. Variable separation and Fourier Analysis. 1D-Resolution. The Laplace and Poisson Equation in bounded domains (at least in one coordinate). Elliptic variants. Variable separation and Fourier Analysis. 2D-Resolution in simple geometries in cartesian coordinates. The 2D eigenvalue problem. Sturm-Liouville problems for self adjoint operators. The Laplace and Poisson Equation in bounded domains (at least in one coordinate). Variable separation and Fourier Analysis. 2D-Resolution in simple geometries in polar coordinates. (disks, rings).

6 ECTS credits.

4

Fourth Course

4.1 First Semester

4.1.1 Numerical Methods

Numerical approximation: introduction; basic concepts; numbers and operations; interpolation; derivation; integration; nonlinear equations; linear systems. Numerical methods for differential equations: discretisation; error; convergence; basic approximations; finite differences for ordinary differential equations; finite differences for partial differential equations.

6 ECTS credits.

4.1.2 Optimization and Network Analysis

Non Linear Optimization (convex optimization and KKT conditions). Introduction to Graphs. Network Optimization (minimum spacing tree, shortest path problem, flow problems, routing). Models and algorithms

6 ECTS credits.

4.1.3 Statistical Models for Prediction

Simple linear regression model. Test about parameters, predictions. Diagnosis and remedial measures: residual analysis; transformations. Multiple regression model. Model, estimation, hypothesis test about parameters. Predictions. Diagnosis. Influence observations, multicollinearity. Variable selection. Criteria for variable selection. Procedures to automatic search. Model validation. Generalized linear models. Exponential family. Logit and Poisson models. Predictions. Hypothesis tests about parameters. Diagnosis, goodness of fit of models.

6 ECTS credits.

4.1.4 Real Analysis

Measure Theory and Integration, Hilbert spaces, Fourier analysis.

6 ECTS credits.

4.1.5 Data Mining

Introduction. Data preparation. Dimensionality reduction techniques. Unsupervised learning techniques: cluster analysis (k.means, hierarchical clustering). Supervised learning techniques: basic techniques, k-NN, decision trees, support vector machines. New trends.

6 ECTS credits.